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Nonlinear super-W algebras at fixed central charge

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We discuss how a class of nonlinear higher-spin superalgebras, containing a Virasoro subalgebra at fixed central charge, can be obtained from a realisation of the super-$W_\infty(\lambda)$ algebra in terms of a supersymmetric BC system. We explicitly work out the example of the nonlinear super-W_2 algebra.

1. Introduction

Higher-spin extensions of the Virasoro algebra play an important role in the study of conformal field theory. Examples of such extensions are the W_N algebras which contain generators of conformal spin $s=2, 3, ..., N$ [1,2]. The W_N algebras have been studied from a variety of viewpoints (see, for instance, refs. [3–6]) [#1]. A characteristic feature of these algebras is that they are nonlinear. The operator product expansion (OPE) of two generators in general contains operators of spin higher than N. Closure is achieved because the operators take the form of (normal ordered) products of the spin $s \leqslant N$ generators.

Another example of a higher-spin extension of the Virasoro algebra is provided by the W_∞ algebra [8,9]. This algebra contains an infinite number of higher-spin generators of conformal spin $s=2, 3, ..., \infty$. In contrast to the W_N algebras the W_∞ algebra is a linear algebra. Because of the infinite number of higher-spin generators there is never the need to write a higher-spin operator, occurring at the RHS of an OPE, as the product of lower-spin generators. Instead, the higher-spin operator is interpreted as a fundamental generator of the W_∞ algebra.

Recently, it has been pointed out that there is an interesting relationship between the W_N and W_∞ algebras [10]. Starting from a $c=-2$ realisation of the W_∞ algebra in terms of a complex free fermion, the authors of ref. [10] derived general expressions for the structure constants of the $c=-2$ W_N algebras. The basic trick is to reinterpret operators which are considered to be fundamental from the viewpoint of the W_∞ algebra as composite operators in the context of the W_N algebras. The situation is similar to what happens in the case of the classical versions of the W_∞ and W_N algebras [6]. The same calculation goes through after bosonising the fermionic realisation of W_∞, as in ref. [11]. For $N=3$ this was done in ref. [12]. The resulting one-scalar realisation of W_3 was first given in ref. [13].

It is natural to consider supersymmetric versions of the W algebras. Higher-spin extensions of the super-Virasoro algebra with a finite number of generators have been studied extensively (see, e.g., refs. [14–20,7]). It turns out that it is nontrivial to construct a super-W_N algebra for arbitrary values of the central charge. In general, the superalgebra can only be defined for specific values of c. This is in contrast to the supersymmetric version of the W_∞ algebra, constructed in ref. [21], which can be defined for arbitrary c.

In this letter we will use the supersymmetric version of the W_∞ algebra to get information on the structure of the super-W_N algebras. We will apply the same techniques as in ref. [10], appropriately adapted to the supersym-

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[#1] For a recent useful review with an extensive list of references, see ref. [7].

metric case. In particular, we will work out explicitly the example of the super-W_2 algebra.

2. The super-$W_\infty(\lambda)$ algebra

Our starting point is the super-$W_\infty(\lambda)$ algebra of ref. [22]. Here λ refers to a one-parameter family of bases of the algebra. It is possible to define a field-theoretic representation of the super-$W_\infty(\lambda)$ algebra in terms of a superconformal BC system. In this representation we have two conformal superfields, a commuting field B and an anticommuting field C, with conformal weights λ and $\frac{1}{2}-\lambda$, respectively. The supersymmetric action equals [23]

$$S=\frac{1}{\pi}\int \mathrm{d}^2z\,\mathrm{d}^2\theta\, B\,\bar{\mathrm{D}}C\,. \tag{1}$$

The generators of the super-$W_\infty(\lambda)$ algebra are related to the following conserved supercurrents of (quasi-)conformal spin $s=\frac{1}{2}, 1, \frac{3}{2}, 2, ...$:

$$W_\lambda^{(s)}=\sum_{i=0}^{2s-1} A^i(s,\lambda)(\mathrm{D}^iB)(\mathrm{D}^{2s-i-1}C)\,, \tag{2}$$

where $\mathrm{D}=\partial_\theta-\theta\partial$ and $\mathrm{D}^2=-\partial$. Normal ordering with respect to the modes of B, C is understood. Note that the supercurrent $W_\lambda^{(s)}$ is commuting (anticommuting) for integer (half-integer) s. The summation index i only takes integer values. Each supercurrent contains a spin s and a spin $s+\frac{1}{2}$ component current. The coefficients $A^i(s,\lambda)$ are given by [22]

$$A^i(s,\lambda)=(1+|2s|_2\,|i+1|_2)\,\frac{(-)^{[s]+[i/2]+|2s+1|_2\,|i+1|_2}}{(-[-s])_{[s]-|2s|_2\,|i|_2}}\binom{[s]-1+|2s|_2\,|i+1|_2}{[i/2]}$$
$$\times(2\lambda-[s])_{[i/2]+|2s+1|_2\,|i|_2}\,(-2\lambda-[s]+1)_{[s]-[i/2]-|i|_2}\,, \tag{3}$$

where $(a)_n\equiv(a+n-1)!/(a-1)!$ and $[a]$ denotes the integer part of a. Furthermore, we have defined $|a|_2\equiv a-2[a/2]$, which is equal to 0 for a even and 1 for a odd. The coefficients given above are determined by the requirement that the supercurrents $W_\lambda^{(s)}$ form $N=1$ superfields with respect to the osp(1, 2) subalgebra of the super-Virasoro algebra which is generated by $W_\lambda^{(3/2)}$.

The first few supercurrents take the form

$$W_\lambda^{(1/2)}=2BC\,,$$
$$W_\lambda^{(1)}=(1-2\lambda)(\mathrm{D}B)C-2\lambda B(\mathrm{D}C)\,,$$
$$W_\lambda^{(3/2)}=(1-2\lambda)(\partial B)C-(\mathrm{D}B)(\mathrm{D}C)-2\lambda B(\partial C)\,,$$
$$W_\lambda^{(2)}=\tfrac{1}{3}(1-2\lambda)(1-\lambda)(\partial\mathrm{D}B)C-\tfrac{1}{3}(1+2\lambda)(1-\lambda)(\partial B)(\mathrm{D}C)$$
$$-\tfrac{1}{3}(1+2\lambda)(1-\lambda)(\mathrm{D}B)(\partial C)+\tfrac{1}{3}\lambda(1+2\lambda)B(\partial\mathrm{D}C)\,,$$
$$W_\lambda^{(5/2)}=\tfrac{1}{3}(1-2\lambda)(1-\lambda)(\partial^2B)C-\tfrac{2}{3}(1-\lambda)(\partial\mathrm{D}B)(\mathrm{D}C)-\tfrac{2}{3}(1+2\lambda)(1-\lambda)(\partial B)(\partial C)$$
$$+\tfrac{1}{3}(1+2\lambda)(\mathrm{D}B)(\partial\mathrm{D}C)+\tfrac{1}{3}\lambda(1+2\lambda)B(\partial^2C)\,,$$
$$\vdots \tag{4}$$

It turns out that the currents $\{W_\lambda^{(1)}, W_\lambda^{(3/2)}\}$ form an $N=2$ super-Virasoro algebra [23]. All currents fit into $N=2$ supermultiplets with respect to the osp(2, 2) subalgebra of this $N=2$ super-Virasoro algebra. The resulting combinations are $\{W_\lambda^{(s)}, W_\lambda^{(s+1/2)}\}$ for integer s and $W_\lambda^{(1/2)}$, where $W_\lambda^{1/1}$ constitutes a so-called $N=2$ scalar multiplet.

The operator product of the superfields $B(z,\theta)$ and $C(z,\theta)$ is equal to [23]

$$C(z_1,\theta_1)B(z_2,\theta_2)\sim\frac{\theta_{12}}{z_{12}}, \tag{5}$$

with $\theta_{12}=\theta_1-\theta_2$ and $z_{12}=z_1-z_2+\theta_1\theta_2$. The $N=1$ super-Virasoro algebra generated by $W_\lambda^{(3/2)}$ is defined by the following operator product expansion:

$$W_\lambda^{(3/2)}(1)W_\lambda^{(3/2)}(2)\sim 3\frac{\theta_{12}W_\lambda^{(3/2)}}{z_{12}^2}-\frac{D_2W_\lambda^{(3/2)}}{z_{12}}+2\frac{\theta_{12}\,\partial_2W_\lambda^{(3/2)}}{z_{12}}-\frac{2(1-4\lambda)}{z_{12}^3}+\text{regular}. \tag{6}$$

The OPE expansion of two general supercurrents $W_\lambda^{(s)}$ and $W_\lambda^{(t)}$ is given by the following expression:

$$W_\lambda^{(s)}(1)W_\lambda^{(t)}(2)\sim\sum_{u=1/2}^{s+t-1/2}f_{st}^u(D_1,D_2;\lambda)\frac{\theta_{12}W_\lambda^{(s+t-u)}(2)}{z_{12}}+c(s,t;\lambda)\frac{\theta_{12}^{|2(s+t)|2}}{(z_{12})^{s+t+|2(s+t)|2/2}}. \tag{7}$$

The structure functions $f_{st}^u(D_1,D_2;\lambda)$ are polynomials in the supercovariant derivatives of degree $2u-1$:

$$f_{st}^u(D_1,D_2;\lambda)=f_{st}^u(\lambda)\sum_{i=0}^{2u-1}M_{st}^u(i)D_1^iD_2^{2u-1-i}. \tag{8}$$

The functions $M_{st}^u(i)$ are completely fixed by the requirement of osp(1, 2) covariance. The structure constants $f_{st}^u(\lambda)$ can be explicitly computed. Since the expressions of $M_{st}^u(i)$ and $f_{st}^u(\lambda)$ are not very illuminating we will refrain from giving them here. They are given by eqs. (4.4) and (4.19) of ref. [22], respectively. Also for the expressions of the central charge $c(s,t;\lambda)$ we refer to ref. [22].

3. The super-W_2 algebra

In order to obtain a nonlinear superalgebra out of the super-$W_\infty(\lambda)$ algebra one should be able to express some of the higher-spin generators with spin $s\geq\frac{5}{2}$ in terms of an appropriately normal-ordered product of the lower-spin generators. We require here a normal-ordering with respect to the modes of the currents themselves. The product of two supercurrents $W_\lambda^{(s)}$ and $W_\lambda^{(t)}$, normal ordered with respect to the modes of $W_\lambda^{(s)}$ and $W_\lambda^{(t)}$, which we denote by $(W_\lambda^{(s)}W_\lambda^{(t)})$, may be defined by [3]

$$(W_\lambda^{(s)}W_\lambda^{(t)})(2)=\frac{1}{2\pi i}\oint dz_1\,d\theta_1\,\frac{\theta_{12}}{z_{12}}\,W_\lambda^{(s)}(1)W_\lambda^{(t)}(2). \tag{9}$$

We will now demonstrate by working out an explicit example how indeed one can obtain nonlinear superalgebras, at fixed central charge, from the super-$W_\infty(\lambda)$ algebra. The example we consider here involves the super-W_2 algebra of refs. [16,18] [#2]. We will remark on the case of more general nonlinear superalgebras in the discussion.

To obtain the super-W_2 algebra of refs. [16,18], we first observe that this superalgebra is generated by the $s=\frac{3}{2}$ Viraso supercurrent and an additional $s=2$ supercurrent. Since there is no $s=1$ supercurrent the algebra has $N=1$ supersymmetry. On the other hand the super-$W_\infty(\lambda)$ algebra has $N=2$ supersymmetry. It turns out that a truncation to $N=1$ supersymmetry is possible for $\lambda=0$ [21,22]. For that value of λ two special things happen. First of all, it turns out that for $\lambda=0$ one can truncate away the $s=\frac{1}{2}$ generator [22]. Second, for $\lambda=0$ the superfields C and DB both have conformal weight $s=\frac{1}{2}$. The truncation to $N=1$ supersymmetry can then be implemented by identifying C with DB.

$$C\equiv DB. \tag{10}$$

[#2] The super-W_2 algebra with zero self-coupling was constructed in ref. [14]. The classical version of the super-W_2 algebra was constructed from the supersymmetric Toda theory corresponding to osp(3|2) in ref. [17].

The effect of this identification is that all currents $W_0^{(s)}$ with s or $s+\frac{1}{2}$ odd vanish identically. The first few nonzero supercurrents are now given by

$$
\begin{aligned}
&W_0^{(3/2)} = 2(\partial B)(\mathrm{D}B)\,,\\
&W_0^{(2)} = \tfrac{2}{3}(\partial \mathrm{D}B)(\mathrm{D}B) + \tfrac{1}{3}(\partial B)^2\,,\\
&W_0^{(7/2)} = \tfrac{1}{5}(\partial^3 B)(\mathrm{D}B) + \tfrac{3}{5}(\partial^2 \mathrm{D}B)(\partial B) - \tfrac{6}{5}(\partial^2 B)(\partial \mathrm{D}B)\,,\\
&W_0^{(4)} = \tfrac{2}{35}(\partial^3 \mathrm{D}B)(\mathrm{D}B) + \tfrac{6}{35}(\partial^3 B)(\partial B) - \tfrac{18}{35}(\partial^2 \mathrm{D}B)(\partial \mathrm{D}B) - \tfrac{9}{35}(\partial^2 B)^2\,,\\
&\vdots
\end{aligned}
\tag{11}
$$

We next consider the OPEs between the $s=\frac{3}{2}$ and $s=2$ supercurrents. In particular we want to investigate which new generators occur at the RHS of these OPEs and whether they can be written as products of the $s=\frac{3}{2}$ and $s=2$ generators. The basic operator product expansion of the B superfield is given by

$$
\mathrm{D}_1 B(z_1,\theta_1)B(z_2,\theta_2) \sim \frac{1}{2}\frac{\theta_{12}}{z_{12}}\,.
\tag{12}
$$

The OPE between two $s=\frac{3}{2}$ supercurrents is given by $N=1$ super-Virasoro algebra for fixed central charge $c(\frac{3}{2},\frac{3}{2};0)=-1$ (see eqs. (6) and (7)) [#3]. The OPE between an $s=\frac{3}{2}$ and an $s=2$ supercurrent is given by

$$
W_0^{(3/2)}(1)\,W_0^{(2)}(2) \sim 4\frac{\theta_{12}W_0^{(2)}}{z_{12}^2} - \frac{\mathrm{D}_2 W_0^{(2)}}{z_{12}} + 2\frac{\theta_{12}\,\partial_2 W_0^{(2)}}{z_{12}} + \text{regular}\,.
\tag{13}
$$

This OPE tells us that $W_0^{(2)}$ is a primary superfield with conformal weight $s=2$.

The last and most interesting OPE to consider is the one between two $s=2$ supercurrents. According to the super-$\mathrm{W}_\infty(0)$ algebra this OPE is given by

$$
\begin{aligned}
W_0^{(2)}(1)\,W_0^{(2)}(2) \sim{}& -\frac{2}{3}\frac{\theta_{12}W_0^{(3/2)}}{z_{12}^3} + \frac{2}{9}\frac{\mathrm{D}_2 W_0^{(3/2)}}{z_{12}^2} - \frac{4}{9}\frac{\theta_{12}\,\partial_2 W_0^{(3/2)}}{z_{12}^2} + \frac{1}{9}\frac{\partial_2 \mathrm{D}_2 W_0^{(3/2)}}{z_{12}} - \frac{1}{6}\frac{\theta_{12}\,\partial_2^2 W_0^{(3/2)}}{z_{12}}\\
&+ \frac{2}{3}\frac{W_0^{(2)}}{z_{12}^2} + \frac{1}{3}\frac{\theta_{12}\mathrm{D}_2 W_0^{(2)}}{z_{12}^2} + \frac{1}{3}\frac{\partial_2 W_0^{(2)}}{z_{12}} + \frac{1}{5}\frac{\theta_{12}\,\partial_2 \mathrm{D}_2 W_0^{(2)}}{z_{12}} - \frac{\theta_{12}W_0^{(7/2)}}{z_{12}} + \frac{\frac{1}{6}}{z_{12}^4} + \text{regular}\,.
\end{aligned}
\tag{14}
$$

From eq. (14) we see that only one new generator occurs at the right-hand side. This is the $s=\frac{7}{2}$ supercurrents $W_0^{(7/2)}$. The question is now whether we are able to express the $s=\frac{7}{2}$ supercurrent in terms of normal-ordered products of the $s=\frac{3}{2}$ and $s=2$ supercurrents and derivatives of these currents. We find that there is indeed a unique decomposition which is given by

$$
W_0^{(7/2)} = \tfrac{2}{9}(\mathrm{D}W_0^{(3/2)}W_0^{(3/2)}) - \tfrac{1}{6}\partial^2 W_0^{(3/2)} + \tfrac{4}{3}(W_0^{(3/2)}W_0^{(2)}) + \tfrac{8}{15}\partial \mathrm{D}W_0^{(2)}\,.
\tag{15}
$$

We have chosen here a certain order for the normal-ordered products. This is no restriction since differently ordered products differ from each other by derivatives of lower-spin currents. A new feature that occurs in the decomposition (15) is the occurrence of the superspace derivative in the nonlinear terms at the RHS of the equation. This is different from the bosonic case where a decomposition can always be achieved without using explicit derivatives in the nonlinear terms [10].

The above calculation shows that the $W_0^{(3/2)}$ and $W_0^{(2)}$ supercurrents, using the decomposition (15), generate a closed algebra. This nonlinear superalgebra is exactly the super-W_2 algebra constructed in refs. [16,18]. To make further contact with the component formulation of refs. [16,7], we expand the $s=\frac{3}{2}$, 2 and the composite $s=\frac{7}{2}$ supercurrents in components as follows:

[#3] Note that the central charge is twice smaller than the one given by eq. (6) for $\lambda=0$. This is due to the fact that the truncation (10) reduces the degrees of freedom by a factor 2.

$$W_0^{(3/2)} = \mathrm{i}G + 2\theta T\,, \quad W_0^{(2)} = -\tfrac{1}{3}\sqrt{2}(W - \sqrt{5}\,\theta U)\,, \quad W_0^{(7/2)} = \Phi^{(7/2)} + \theta\Phi^{(4)}\,. \tag{16}$$

This leads to the following component version of the OPE between two $s=2$ supercurrents:

$$W(1)W(2) \sim \frac{2\{T - \frac{1}{2}\sqrt{2}\,W\}}{(z_1 - z_2)^2} + \frac{\partial_2\{T - \frac{1}{2}\sqrt{2}\,W\}}{z_1 - z_2} + \frac{\frac{3}{4}}{(z_1 - z_2)^4}\,,$$

$$U(1)W(2) \sim \frac{\frac{3}{5}\mathrm{i}\sqrt{5}\,G}{(z_1 - z_2)^3} + \frac{\frac{2}{5}\mathrm{i}\sqrt{5}\,\partial_2 G}{(z_1 - z_2)^2} + \frac{\frac{3}{20}\mathrm{i}\sqrt{5}\,\partial_2^2 G}{z_1 - z_2} - \frac{\frac{1}{2}\sqrt{2}\,U}{(z_1 - z_2)^2} - \frac{\frac{3}{10}\sqrt{2}\,\partial_2 U}{z_1 - z_2} + \frac{\frac{9}{10}\sqrt{5}\,\Phi^{(7/2)}}{z_1 - z_2}\,, \tag{17}$$

$$U(1)U(2) \sim \frac{2\{T - \frac{1}{5}\sqrt{2}\,W\}}{(z_1 - z_2)^3} + \frac{\partial_2\{T - \frac{1}{5}\sqrt{2}\,W\}}{(z_1 - z_2)^2} + \frac{\frac{3}{10}\partial_2^2\{T - \frac{1}{5}\sqrt{2}\,W\}}{z_1 - z_2} + \frac{\frac{9}{10}\Phi^{(4)}}{z_1 - z_2} + \frac{\frac{3}{5}}{(z_1 - z_2)^5}\,, \tag{18}$$

with the composite fields $\Phi^{(7/2)}$ and $\Phi^{(4)}$ given by

$$\Phi^{(7/2)} = \tfrac{4}{9}\mathrm{i}(TG) - \tfrac{4}{9}\mathrm{i}\sqrt{2}(GW) + \tfrac{8}{45}\sqrt{10}\,\partial U - \tfrac{1}{6}\mathrm{i}\,\partial^2 G\,,$$

$$\Phi^{(4)} = \tfrac{2}{9}(\partial GG) - \tfrac{4}{9}\mathrm{i}\sqrt{10}(GU) + \tfrac{8}{9}(TT) - \tfrac{8}{9}\sqrt{2}(TW) + \tfrac{8}{45}\sqrt{2}\,\partial^2 W - \tfrac{1}{3}\partial^2 T\,. \tag{19}$$

This result coincides with the results of ref. [16,7] with the central charge c of the Virasoro subalgebra given by $c = -\frac{3}{2}c\,(\frac{3}{2}, \frac{3}{2}; 0) = \frac{3}{2}$ and with the coupling $\mathscr{C}_{22}{}^2(c)$ given by $\mathscr{C}_{22}{}^2(\frac{3}{2}) = -\sqrt{2}$. We should stress that the one scalar superfield realisation constructed here in itself is not new. This realisation is already contained in the two scalar superfield realisation of refs. [16,7,18] with the background charge and one of the scalar superfields set equal to zero.

4. Discussion

In this letter we have shown how to obtain the nonlinear super-W_2 algebra, with central charge c of the Virasoro subalgebra given by $c = \frac{3}{2}$, from the linear super-$W_\infty(\lambda)$ algebra. The fixed central charge came about because we were working with a specific representation of the superalgebra in terms of one real scalar superfield B. An advantage of this approach is that the closure of the (nonlinear) superalgebra is manifest. A restriction of this method is the fixed central charge and the fact that we only consider superalgebras that allow a realisation in terms of a superconformal BC system.

It is natural to consider more general representations (including more superfields) leading to arbitrary values of the central charge [#4]. In fact, in the case of the super-W_2 algebra this has already been done [16,7,18]. It turns out that one needs a representation involving two superfields, one of them with a nonzero background charge. The resulting changes in the algebra are rather simple. The final answer is given by [16,7,18]

$$W_0^{(3/2)}(1)W_0^{(3/2)}(2) \sim 3\frac{\theta_{12} W_0^{(3/2)}}{z_{12}^2} - \frac{\mathrm{D}_2 W_0^{(3/2)}}{z_{12}} + 2\frac{\theta_{12}\,\partial_2 W_0^{(3/2)}}{z_{12}} - \frac{2}{3}\frac{c}{z_{12}^3} + \text{regular}\,,$$

$$W_0^{(3/2)}(1)W_0^{(2)}(2) \sim 4\frac{\theta_{12} W_0^{(2)}}{z_{12}^2} - \frac{\mathrm{D}_2 W_0^{(2)}}{z_{12}} + 2\frac{\theta_{12}\,\partial_2 W_0^{(2)}}{z_{12}} + \text{regular}\,, \tag{20}$$

[#4] For a discussion on this point in the case of the bosonic W algebras, see ref. [24]. Multi-scalar realisations have also been considered in ref. [11].

$$W_0^{(2)}(1)W_0^{(2)}(2)\sim-\frac{2}{3}\frac{\theta_{12}W_0^{(3/2)}}{z_{12}^3}+\frac{2}{9}\frac{D_2W_0^{(3/2)}}{z_{12}^2}-\frac{4}{9}\frac{\theta_{12}\partial_2W_0^{(3/2)}}{z_{12}^2}+\frac{1}{9}\frac{\partial_2D_2W_0^{(3/2)}}{z_{12}}-\frac{1}{6}\frac{\theta_{12}\partial_2^2W_0^{(3/2)}}{z_{12}}$$
$$-\tfrac{1}{2}\sqrt{2}\,\mathscr{C}_{22}{}^2\left(\frac{2}{3}\frac{W_0^{(2)}}{z_{12}^2}+\frac{1}{3}\frac{\theta_{12}D_2W_0^{(2)}}{z_{12}^2}+\frac{1}{3}\frac{\partial_2W_0^{(2)}}{z_{12}}+\frac{1}{5}\frac{\theta_{12}\partial_2D_2W_0^{(2)}}{z_{12}}\right)-\frac{\theta_{12}W_0^{(7/2)}}{z_{12}}+\frac{1}{9}\frac{c}{z_{12}^4}$$
$$+\text{regular}\,, \qquad (20\text{ cont'd})$$

with the self-coupling $\mathscr{C}_{22}{}^2$ and the composite $s=\frac{7}{2}$ operator $W_0^{(7/2)}$ given by

$$(\mathscr{C}_{22}{}^2)^2=\frac{4(5c+6)^2}{(4c+21)(15-c)} \qquad (21)$$

and

$$W_0^{(7/2)}=+\frac{6}{4c+21}\left[(DW_0^{(3/2)}W_0^{(3/2)})-\tfrac{3}{4}\partial^2W_0^{(3/2)}\right]-\frac{9\sqrt{2}\,\mathscr{C}_{22}{}^2}{5c+6}\left[(W_0^{(3/2)}W_0^{(2)})+\tfrac{2}{5}\partial DW_0^{(2)}\right] \qquad (22)$$

respectively. It remains to be investigated whether one can rederive this result starting from a multi-scalar superfield realisation of the super-$W_\infty(\lambda)$ algebra.

It seems reasonable to expect that the construction carried out in this letter can be extended to nonlinear superalgebras involving more higher-spin generators. The super-W_2 algebra is the most simple example of a nonlinear superalgebra with $N=1$ supersymmetry. The next simple example would be an $N=1$ superalgebra generated by an $s=\frac{3}{2}$, 2 and $s=\frac{7}{2}$ supercurrent [#5].

It would be interesting to investigate in more detail the case where the nonlinear superalgebra contains a bosonic $s=3$ generator since such an algebra is expected to contain W_3 as a bosonic subalgebra [#6]. From the point of view of the super-$W_\infty(\lambda)$ algebra, one should retain either the $s=\frac{5}{2}$ or the $s=3$ supercurrent since these are the only supercurrents that contain a bosonic $s=3$ generator. One should therefore not perform the truncation $C\equiv DB$ (see after eq. (10)) and we expect an algebra with $N=2$ supersymmetry. The simplest possibility (containing only one bosonic spin 3 generator) is an $N=2$ supersymmetric extension of the super-W_2 algebra. This extension would contain the $\{W_0^{(1)}, W_0^{(3/2)}\}$ and the $\{W_0^{(2)}, W_0^{(5/2)}\}$ $N=2$ supermultiplets as its generators. We note that this is precisely the field content of the super-W_3 algebra considered in ref. [20] and of the $N=2$ super-W_2 algebra mentioned in refs. [17,25].

Finally, we should mention that in general the supercurrents generating the super-$W_\infty(\lambda)$ algebra are quasi-primary fields. Only in exceptional cases, like for the super-W_2 algebra, the relevant supercurrents are primary. It would be interesting to see whether in the nonlinear case the quasi-primary fields can be made primary after some field redefinitions or whether one should allow such quasi-primary fields to be part of the algebra.

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[#5] This algebra seems to occur in the context of an osp(3|4) Toda theory as mentioned in the tables of refs. [18,25].
[#6] For earlier work on supersymmetric W_3 algebras, see refs. [14,15,17,19,20,25,26].

Note added

After the completion of this work I received a preprint [27] where an $=2$ super-W_3 algebra at arbitrary central charge with the field content metioned in the discussion is constructed.

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